ACHIEVEMENTS OF CONTEMPORARY BHARAT

सत्यमेव जयते
MINISTRY
OF EDUCATION
Government of India

CHANDRAYAAN UTSAV

CODE
1.10HS

SPECIAL MODULE

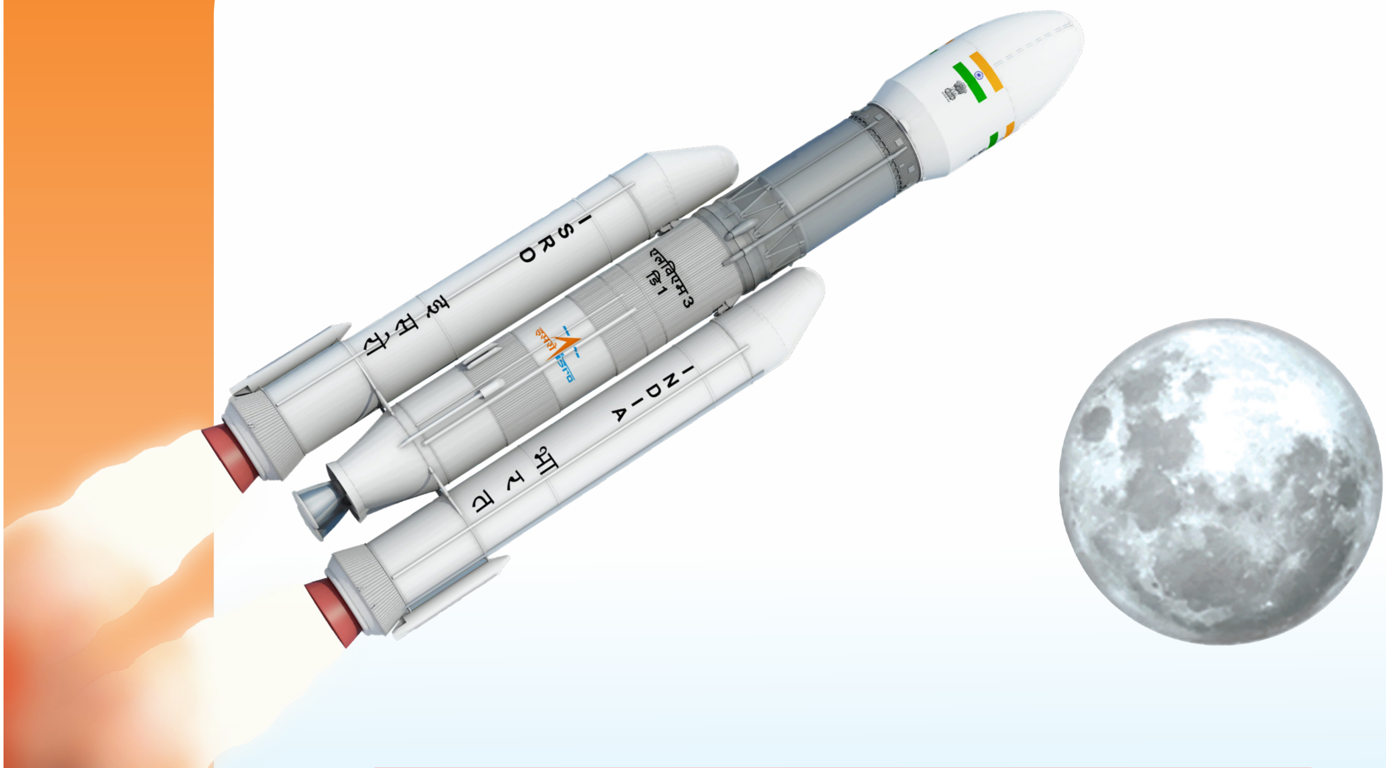
विद्यया ऽमृतमश्नुते
एन सी ई आर टी
NCERT

17-10-2023 9.05.23 PM

October 2023
Ashwina 1945

PD 1T BS

Published at the Publication Division, by the Secretary, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 110 016 and printed by Gita Offset Printers (P.) Ltd., C-90 & C-86, Okhla Industrial Area, Phase-I, New Delhi 110 020



Physics of Chandrayaan-3

Higher Secondary Stage

Let us embark on an exhilarating journey into the depths of space exploration and celestial mechanics, with a focus on India's remarkable lunar mission, Chandrayaan-3. Our voyage will take us through fundamental scientific principles, cutting-edge technology, and the awe-inspiring process of soft landing on the Moon's enigmatic South Pole.

In Indian philosophy, the pursuit of knowledge is highly revered, and Chandrayaan-3 embodies this value by advancing our understanding of the Moon and Space. By exploring the mission's scientific aspects, students can appreciate the Indian tradition of seeking knowledge and wisdom. Our ethos orbits around "Vasudhaiva Kutumbakam". The honourable Prime Minister emphasised upon human centric development. Our quest for space is not a part of space race, but our achievements will add to the benefit of the entire humankind. Chandrayaan-3 is a testament to India's multicultural and collaborative approach to science and technology, involving contributions from various regions and communities.

Chandrayaan-3, India's third lunar exploration mission, presents an excellent opportunity to engage students in the realms of Science, Technology, Engineering, and Mathematics (STEM). By delving into this mission, students can explore fundamental physics concepts, such as gravity, motion, propulsion, etc. In addition to its educational merits, it can cultivate critical thinking and problem-solving skills among students by addressing the complex challenges encountered during the mission, including lunar orbit insertion and planned soft landing.

Furthermore, Chandrayaan-3 represents a matter of pride for India, exemplifying the nation's achievements in the field of space science and technology. Learning about this mission can instill a sense of national pride and encourage students to appreciate the importance of scientific research and innovation. Importantly, this module offers a glimpse into various career opportunities in space research, engineering, and astronomy, enabling students to make informed decisions about their future academic and career paths. It also underscores the cross-disciplinary nature of space exploration, encouraging students to recognize how different fields of knowledge intersect and collaborate in real-world scientific endeavors. All of this complies with the National Education Policy 2020's recommendations.

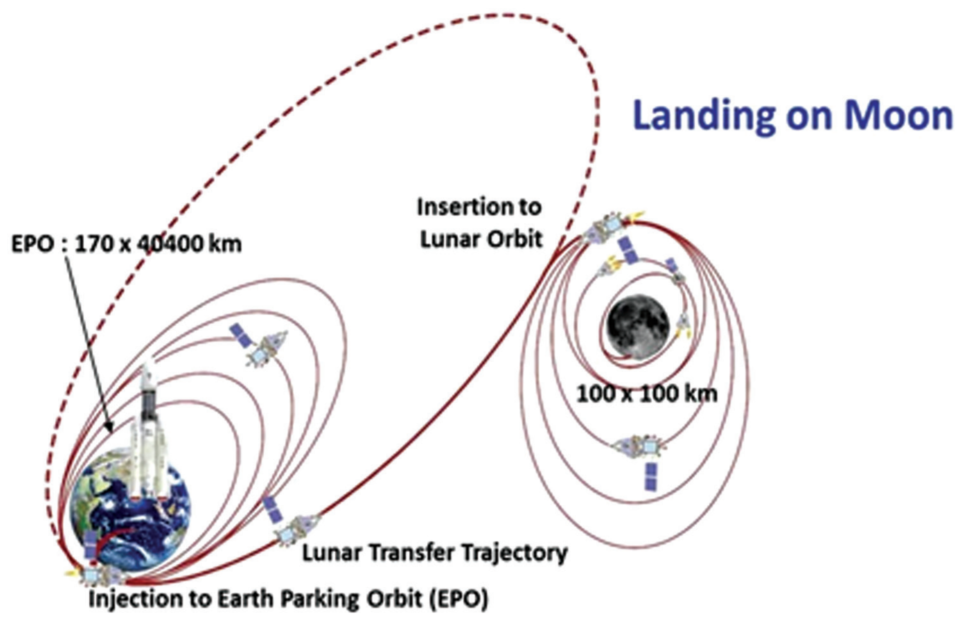
Astrodynamics

The exploration of space and the celestial waltz of planets and stars have always held a mysterious allure for humanity. In the annals of science and discovery, the luminous minds of Galileo Galilee, Johannes Kepler, and Sir Isaac Newton shone brilliantly, illuminating the intricate choreography of the heavens. Their elegant equations and profound insights transcended earthly confines, unveiling the secrets of orbital mechanics, and revealing the cosmic ballet that governs our celestial neighborhood. As we delve deeper into this cosmic odyssey, we cannot help but be inspired by the indomitable spirit of Wernher von Braun (father of rocket science), the visionary engineer who transformed dreams of reaching the stars into tangible rockets that breached Earth's atmosphere. His towering achievements turned the boundless expanse of space into an attainable frontier, where humanity's yearning for exploration could take flight. This is a tale of science and dreams, of equations that span the ages, and of engineers and explorers whose visions propelled us beyond the skies. It's a journey where the poetry of the cosmos meets the precision of engineering, where the pioneers of yesteryears' stargazing have paved the way for today's interstellar pioneers. In this celestial ballet, we find both wonder and the invitation to explore the universe, guided by the brilliance of these luminaries, towards a destiny among the stars.

Astrodynamics is a specialised field within aerospace engineering and physics that focuses on the study of the motion and dynamics of objects in space. It encompasses a wide range of principles and calculations related to the behaviour of spacecraft, satellites, planets, and celestial bodies as they move through the cosmos. Astrodynamics plays a critical role in space mission planning, satellite orbit determination, and spacecraft navigation. It involves understanding the complexities of orbital mechanics, gravitational interactions, propulsion systems, and trajectory planning to ensure the successful execution of space missions. Astrodynamics is not only a fundamental discipline for space exploration but also a key driver of technological advancements that enable humanity to venture deeper into the cosmos.

The Satellite and their motion

India launched Chandrayaan-3 spacecraft using popularly called "Bahubali". Launching Vehicle (LVM3- M4). To keep things simple we will be calling it Rocket. It was launched from Satish Dhawan Space Center, SHAR, Sriharikota, Andhra Pradesh. It possesses vast amount of energy to overcome gravitational pull of the earth. Then it has to periodically increase its speed (maneuvering) till it achieves desirable speed so that it can be sling shot into the gravitational field of the moon. There it has to de-maneuver (periodically decrease its speed) till its orbit is circular to make soft landing to the moon.



To imagine the complexity of the process let's use an analogy. You might have seen velodrome or wall of death or "Maut ka kuan" in carnivals. There a motorcyclists moves along the inner wall of the velodrome without falling. The shape of the velodrome is like a frustum which a sliced cone is leaving its vertex like a bucket. Orbit of the rider is a plane intersecting the axis of the velodrome (axis of the cone). So the rider is moving in an elliptical path (if his orbital plane is oblique to the axis of the velodrome). Now if the rider keeps on increasing his speed periodically, he gains kinetic energy which overcomes the gravitational potential of the earth and rises gradually almost up to the brim of the velodrome. Same is the case for a rocket, if it has to perform Trans-lunar journey then our chandrayaan too is supposed to increase its speed to achieve a desirable speed so that it can reach up to the gravitational field of the moon. Now extending our analogy of the velodrome, let's now imagine that there is another velodrome nearby and the biker has to enter in the neighbouring velodrome and land safely to the floor. As the biker left the previous velodrome with high speed, he/she is supposed to decrease his/her speed so that he/she can hit the floor safely. To increase the complexity of the situation, let's assume that the other velodrome is not stationary but moving around the previous velodrome. Now if the biker ejects out of his velodrome early or late relative to the circling velodrome then he/she is sure to meet the ill fate of crash landing. Same is the case with Chandrayaan-3. Even entering the gravitational field of the moon is too difficult, making safe landing is different thing. So we need to study basic physics involved to ensure soft landing to the south pole of moon.

Activity 1

Take a small plastic bucket (or anything similar). Place a small ball into it, now set the bucket in small circular motion placing it on a smooth surface.

Is the ball moving along the inner walls of the bucket (like a biker in "Maut ka Kuan"?

If the bucket is moved with greater speed, does the orbiting ball in the bucket rise upward?

Does the orbital path of the ball resembles elliptical path (bucket may be tilted a bit while performing circular motion)?

Can you set the ball at a speed so that it lands in a nearby bucket?

Reduced Mass

Let us consider an isolated system consisting of two particles m_1 and m_2 under the attraction of a central force say gravity or Columb force (force acting along the line joining the centers of the particles and depends on the distance between them.

The position vector of m_1 and m_2 is R_1 and R_2 respectively.

Center of mass is the imaginary point along the line joining the centres of the two masses where net force acting on the system is zero (without any turning effect). This point is called barycentre, between two orbiting bodies. If satellite orbiting around a planet is comparable to the planet, then the binary system (say Earth and Moon) revolves around their common barycenter.

The position vector of their centre of mass, RCM = $m_1R_1 + m_2R_2 = 0$ (1)

(Origin can be chosen at center of mass)

Vector joining the two masses, $R = R_1 - R_2$ (2)

$R_2 = (-m_1/m_2)R_1$ (3)

$R = R_1 + (m_1/m_2)R_1 = (1 + m_1/m_2)R_1$ (4) (from (1) and (2))

Force acting on m_1, $m_1 d^2R_1/dt^2 = F_1$ (5)

Force acting on m_2, $m_2 d^2R_2/dt^2 = F_2$ (6)

Using (3) and (4) in (5) and (6)

$(m_1m_2/m_1+m_2)\ d^2R/dt^2 = F_1 = F_2 = \mu\ d^2R/dt^2$ (7)

$\mu = m_1m_2/m_1+m_2$ is called the reduced mass and (7) is the equation of motion (Newton`s second law) of m_1 about m_2. Now we can solve (7) of m_1 relative to m_2 (earth), exactly as though m_2 is fixed and m_1 moving.

Under the influence of gravitational force, $\mu\ d^2R/dt^2 = -Gm_1m_2r/R_2$ (8)

Thus, now we have reduced a two body problem (earth and moon) as one body problem.

Orbital Mechanics

In the present case of Chandrayaan, its mass is negligible compared to the mass of Earth or Moon so there is no need to take reduced mass.

Chandrayaan (having mass m) has both linear (radial speed) and angular speed.

(Motion along a straight line has only linear motion and angular speed is zero. In circular motion, distance of the moving particle from the centre is constant thus its radial speed is zero. But in rocket/satellite dynamics, both its distance from the centre and its angular position keeps changing therefore both radial and angular speeds are non zero)

Radial part of the speed, $v_r = dr/dt$

Angular part of the speed, $v_\Theta = r(d\Theta/dt)$ (9)

So its kinetic energy is $K = m(v_r^2 + v_\Theta^2)/2$

Angular momentum of the satellite under the influence of Gravitational force,

$L = mv_\Theta r = mr^2 d\Theta/dt$.......... (10) (Using (9))

So $m(v_\Theta^2)/2 = m(rd\Theta/dt)^2/2 = L^2/2mr^2$

By the law of conservation of energy, $m(dr/dt)^2/2 + L^2/2mr^2 + V = E$ (11)
(V is the Gravitational potential energy)

Dividing (11) by the square of eq. (10) we have,

$((dr/dt)^2/(r^2 d\Theta/dt))^2 = (2m/L^2)[E - V - (L^2/2mr^2)]$

$(1/r^2)(dr/d\Theta)^2 = 2mE/L^2 - 1/r^2 - 2mV(r)/L^2$

$V(r) = -\beta/r$ (β = GMm, M is the mass of Earth and m is the mass of Chandrayaan. Here negative sign of V(r) implies that force is attractive)

Let's assume bigger body (say Earth) is nailed at the origin.

$(1/r^2)(dr/d\Theta)^2 = 2mE/L^2 - 1/r^2 - 2m\beta/rL^2$ (12)

$d(1/r)/d\Theta = -(1/r^2)dr/d\Theta)$ (13) and assuming $y = 1/r$

From (4) and (5), $(dy/d\Theta)^2 = -y^2 + 2m\beta y/L^2 + 2mE/L^2$

$= -(y - m\beta/L^2)^2 + 2mE/L^2 + (m\beta/L^2)^2$

On substituting, $z = y - m\beta/L^2$

$dz/d\Theta = -z^2 + (m\beta/L^2)^2(1 + 2EL^2/m\beta^2) = -z^2 + B^2$ (14)

Where $B = (m\beta/L^2)[1 + 2EL^2/m\beta^2]^{1/2}$

Using separation of variables in (14),

$$d\theta = \int dz/\sqrt{(B^{\wedge}(2\quad)-Z^{\wedge 2})}$$

$\Theta - \Theta_0 = \cos^{-1}(z/B)$ (Θ_0 is the constant of integration)

$Z = B\cos(\Theta - \Theta_0)$ (on proper choice of the axis, Θ_0 can be set as 0)

$Z = 1/r - m\beta/L^2 = B\cos(\Theta)$

$$(m\beta/L^2)\sqrt{(1+2EL^2/m\beta^2)}(\cos\theta) = 1/r$$

$$1/r = (m\beta/L^2)(1 + e\cos\theta)$$

It's the equation of a conic section. Thus under central force, planets or satellites moves in conic section (hyperbola/ ellipse/ circle/ parabola), Where, e is the eccentricity of the conic section.

$e = [1 + 2EL^2/m\beta^2]^{1/2}$ (eccentricity of the conic section)

Thus, we have arrived at a very important conclusion that, trajectory/orbit of a satellite is a conic section and actual celestial path (elliptical/hyperbolic/circular/parabolic) will depend on its eccentricity and thus energy/speed. It is to be noted that when satellite keep circuiting in the same path then its called its orbit and if it pass just once through any particular path then it is called its trajectory.

General equation of conic section in polar form is, $1/R = (1-e\cos(\theta))/a(1-e^2)$. (a is semi major axis)

$(R_{max} - R_{min})/(R_{max} + R_{min}) = e$ (eccentricity of the ellipse)

Where R_{max} is the maximum distance from the center of the earth called perigee.

Where R_{min} is the minimum distance from the center of the earth called apogee.

Similarly, same physics is applied at atomic level too where electrons are orbiting the nucleus and if you remember α particle scattering (Rutherford experiment) energetic α particle (He^{++}) got scattered from the Gold nucleus following hyperbolic path.

(Why do the subatomic particles follows elliptical and hyperbolic paths in Coloumb force field?)

Activity 2

Take a torch and switch it on. What should be the beam profile of the light coming from the torch?

Illuminate the wall of your room in dark at various angles, what is the shape of the light observed on the wall when the torch is perpendicular to the wall?

What is the shape of the light spot when the torch is oblique (slanted)?

What happens to the light spot if torch is more oblique?

If torch is held parallel to the wall and close to it, what is the shape obtained on the wall?

Can you correlate it with conic section?

If the same process is repeated with a bucket containing water, note your observations regarding free surface of water (shape of boundary of water touching inner walls of the bucket).

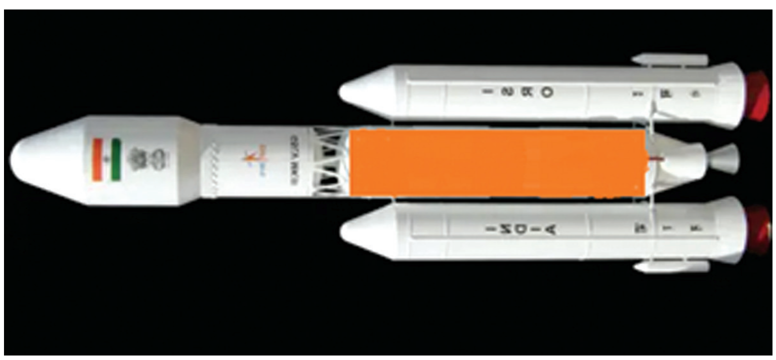
Thus, the equation discussed provides very important information regarding the trajectory/orbit of Chandrayaan-3.

e = 0	E = -μG2m12m22/2L2	Circular orbit (closed path)	Bounded
0 < e < 1	E < 0	Elliptical orbit (closed path)	bounded
e = 1	E = 0	Single parabolic trajectory (open path)	escape
e > 1	E > 0	Hyperbolic trajectories (open path)	escape

Thus, depending on the velocity of Chandrayaan-3, it can have circular or elliptical or parabolic or hyperbolic trajectory/orbit.

Rocket Equation

We need now to explore the interesting Physics behind the motion of the rocket. Let us assume that the instantaneous mass of the rocket including the mass of the fuel is 'm'.



Exhaust gas velocity is zero and so is rocket velocity.

Exhaust gas velocity, $v_g < 0$, Rocket velocity, $v_r > 0$.

(dm_g is the mass of exhaust gas expelled out. dm_r is the decrease in the weight of the rocket), In space, external force on the rocket and the exhaust gas is negligible hence rocket and exhaust gases may be considered as closed system and hence linear momentum must be conserved.

Physics of Chandrayaan 1_10HS.indd 9 10/14/2023 1:41:05 AM

Momentum of rocket = Momentum of exhaust gas,

$dm_g v_g = -m_r dv_r$

($dm_g = dm$ and $m_r = m$ that is, instantaneous mass of the rocket) (15)

$v_g dm = -m dv_r$ (integrating either side after separation of the variables,)

$v_g \int dm/m = -\int dv_r$

$\mathbf{\Delta V_r = (V_g) ln(m_i/m_f)}$

This is called rocket equation

On rewriting the rocket equation, $\Delta v_f = v_g \ln (R_m)$ (v_f is the final rocket velocity, to reduce complexities derivation may be neglected and only the final formula may be retained)

R_m is the mass ratio, $R_m = (M + C)/M$

Where, M is the total mass of the rocket without fuel and C is the mass of the fuel

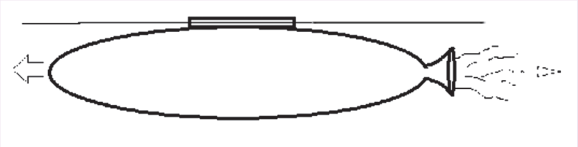
If $v_f = v_g \ln (R_m) = 1$ i.e. $R_m = 2.718$ ($v_i = 0$ so $\Delta v_r = v_f$)

C is 63.2% of total mass.

For $v_f = 2v_g$, R_m should not be doubled but squared (relation is nonlinear and logarithmic)

For the same reason if 5% of fuel is increased then it increases v_f from 1.38 times to 1.6 times if the increase of the fuel is from 75% to 80% and if the fuel is raised again by 5% but from 90% to 95% than v_f increases from 2.3 to 6.9 times, that's again due to no nonlinear logarithmic nature of rocket equation.

Activity 3



Take a cucumber shaped balloon. Inflate it and glue a straw along the axis of the inflated balloon. Now tie a thread on a wall or table and pass the other end of the thread through the straw glued on the inflated balloon. Tie the other end of the thread at suitable place such that the thread remains little tensed. Now take the balloon at the other end and release it. As the pressured air trapped in the balloon gushes out of it, our balloon rocket will move along the thread.

If nozzle of the balloon is reduced than what happens to the speed of the gushing out air and speed of the balloon rocket and why?

Multi Stage Rocket

A rocket may be subdivided into multiple stages for the benefit of attaining high speed which is not possible for a similar single stage rocket. A velocity achieved in each stage is cumulative. To achieve desirable speed, number of stages can be increased. But beyond a stage it's practically impossible to further add the stages because the rocket material is delicate, each additional stage adds to the complexity of assembling. Chandrayaan 3 is a three stage rocket.

$v_f = v_g$ (rocket equation) (in the first stage, $V_i = 0$ but in the subsequent stages, final speed of the last stage will be initial speed for the next stage)

Let $v_f = v_g = 2.65\ kms^{-1}$

Therefore, successive stages will have velocities, 2.650 kms^{-1}, 5.3 kms^{-1} and 7.95 kms^{-1}

(Left as an exercise for the students to calculate above speeds using rocket equation)

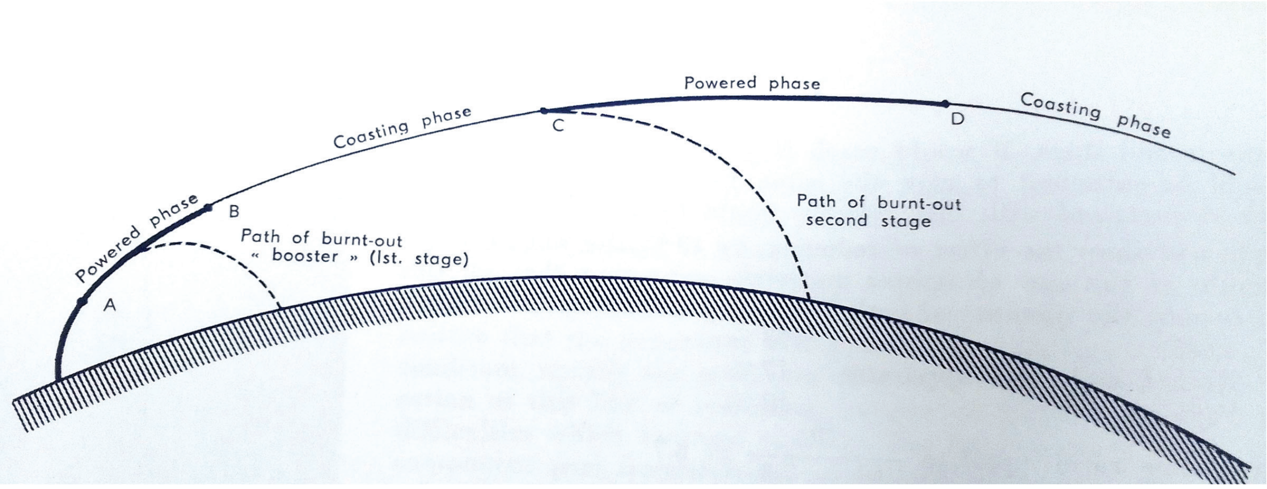
Composition	Stage I	Stage II	Stage III
Fuel	6300kg	630kg	63kg
Rocket Body	2700kg	270kg	27kg
Payload	1000kg	100kg	10kg
Total	10,000kg	1000kg	100kg

Total mass at launch = 10,000kg
Final Payload = 100kg (mass of the third stage)

$v_f = 7.95 kms^{-1}$

But for an identical single stage rocket, R_m = 10 tonnes/(10 tonnes – 7 tonnes) = 3.33

And $v_f = 3.181 kms^{-1}$, which is less than half of speed attained by the three stage rocket.



It should be noted that the first stage needs to provide the maximum thrust as it has to fight against the atmospheric drag. Hence, it would be wise to travel least distance through the thick atmosphere of the earth during the launch. So the Chandrayaan should be fired vertically upward (as you might have seen the launch of the Chandrayaan on TV). But it can't keep moving vertically, it has to steer into its orbit. As the Chandrayaan reaches the upper atmosphere in short time (powered phase) it jettisons the first stage and keeps moving by inertia alone (coasting phase) then at predetermined speed and altitude, it again undergoes ignition of second stage (second powered phase) and subsequently it attains an elliptical orbit in space.

Launch and Stabilisation

To minimize air drag, the Chandrayaan must be launched vertically and after a predetermined altitude, the directional devices come into operation and the trajectory inclined progressively and rocket achieve elliptical trajectory. But when rocket is inclined then there is no braking system in the space, hence a couple will act about the centre of mass of the rocket which may somersault the rocket. Fins may not be of any use in the upper atmosphere or in space as there is hardly any air. So, as the shell or bullet is given an initial turn about its axis, the shell/bullet remains stable due to the gyroscopic effect. Even the olympic gold medalist, Neeraj Chopra imparts an initial angular momentum to his javelin about its axis to stop it from somersaulting. Similarly, there are auxiliary rockets called vernier rockets, fired to stabilise the rocket. Same rockets are used for inclining the rocket as well. Thus Vernier rockets acts as both directional as well as stabilising devices.

Maximum linear speed due to rotation of the Earth, is on equator, as its distance from the axis of rotation is maximum (about $400 ms^{-1}$). Thus, Chandrayaan should be launched along Eastern direction near equator to gain additional speed of rotation of the Earth. Hence the launching pad should be southernmost near to equator.

Maneuvering to Lunar Gravitational field

Chandrayaan after sequenced maneuvering has to achieve certain speed so that it can reach up to the gravitational neutral point of Earth Moon system. It's the imaginary position on the imaginary line joining the centre of Earth-Moon where Intensity of Gravitational field of both Earth and Moon is same.

Mass of earth is M_E, distance of neutral point from Earth, R_E

Mass of Moon is M_M, distance of neutral point from Moon, R_M

$GM_E/R_E^2 = GM_M/R_M^2$ (Geo gravity = Lunar gravity)

$M_E = 81.5\ M_M$

Therefore, $R_E/R_M = 9$

This point is 9/10 of distance of separation of Earth and the Moon. Its distance from Earth is approximately 345000km from the Earth and 38000km from Moon. Distance travelled by the moon in 1 hour is 3600km $>D_m$. (D_m is the diameter of the moon)

Thus, if Chandrayaan reaches this point by an error of 1 hour it will miss the moon by the moon diameter. Even a slight change in the speed of launching, rocket may miss the moon. Propulsion module after safely inserted in the lunar gravitational pull needs to be slowed down. This is achieved by the retro rockets of the propulsion module. By the periodic action of retro rockets, apogee of the propulsion module keeps on decreasing. For the final orbit before soft landing, the propulsion module needs to be anchored in almost circular orbit close to the moon. The propulsion module in coordination with the orbiter of Chandrayaan-3 continuously scanned lunar terrain to identify proper harbouring point at the lunar South Pole. After detachment from the propulsion module, lander module, Vikram housing rover module (Pragyaan) approaches downward descent. Vikram is equipped with state of art cutting edge technology of advanced sensors, velocimeters (horizontal and vertical), cameras and evolved onboard computers and rugged electronics eyeing on the safe landing location. All the descent is controlled by the onboard computers, ground stations is just receiving the velocities and position of the Vikram without any control on landing. Finally, India became the fourth Nation to land on Moon and first Nation to demonstrate soft landing near the South Pole of the Moon (nearly 70° south lunar latitude). This landing point is named as "Shiv Shakti" by the honourable Prime Minister Shri Narendra Modi.

Propellant Physics

You might have studied the process of combustion. It's the exothermic reaction (heat evolving) in the presence of oxygen. In the terrestrial conditions, atmospheric oxygen is utilized for the combustion process like a jet engine. But in the case of voyage to the space, oxygen is not available in the outer space.

Combination of fuel and oxidizer is called propellant. Referring to the rocket equation, for high velocity of the rocket, velocity of the exhaust gas should be very high. If a footballer kicks light and heavy football by same energy, it's easy to figure out that lighter ball will cover longer ground and higher speed. Thus if exhaust gas is lighter than speed of lighter exhaust gas would be high. Hence high calorific value (high calorific valued fuel will yield high temperature and thus exhaust gasses will have greater Kinetic energies and velocities, $V_{RMS} = (3K_BT/2M)^{1/2}$ where M is the molecular mass of the exhaust gases) lighter combustion mass is preferred. Unfortunately the fuels which give rise to lighter combustion products are of low densities which possess a serious problem of huge voluminous fuel storing tanks..

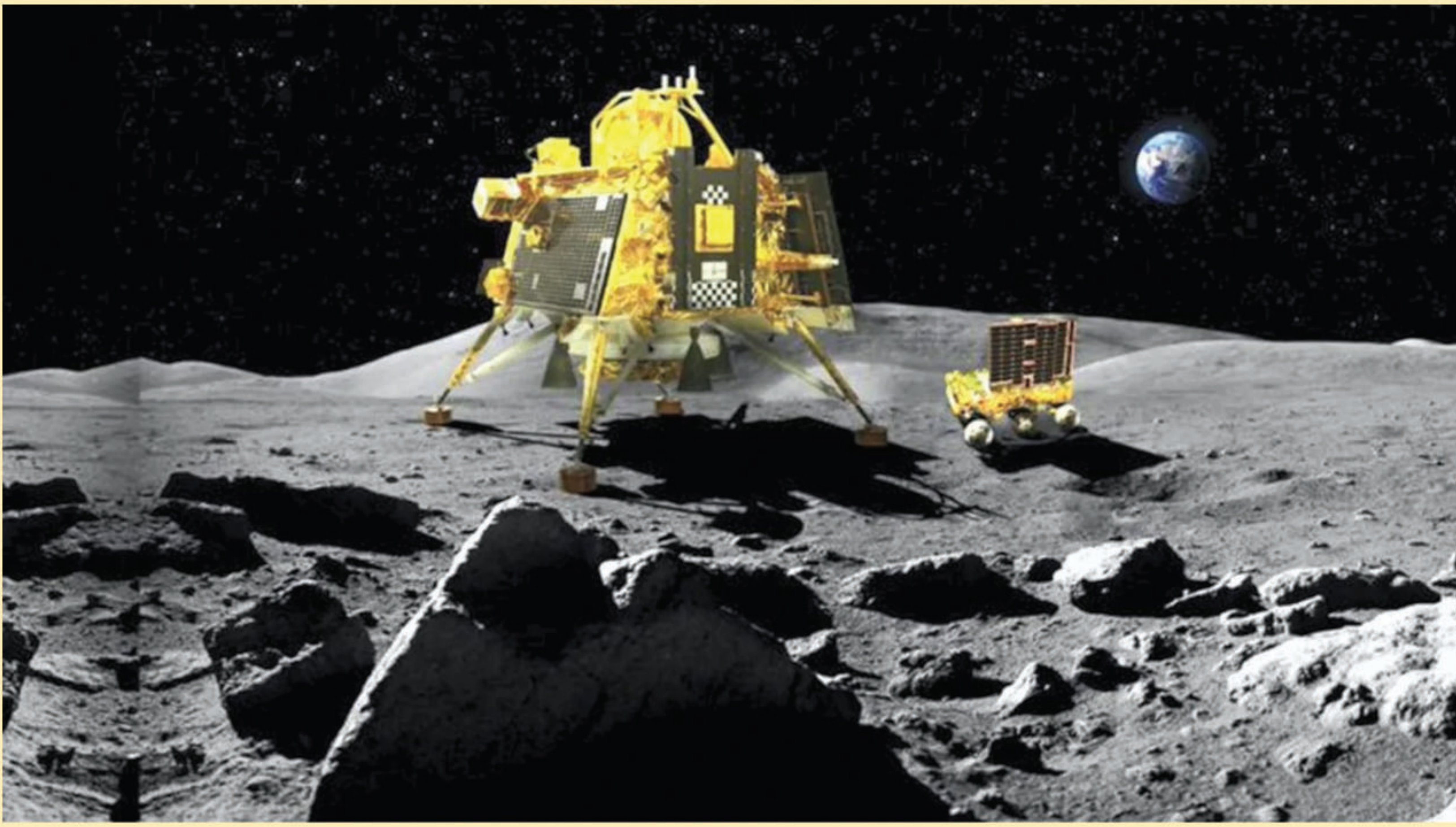
The LVM3 is a three stage rocket. Its first stage uses solid fuel. Solid fuel imparts tremendous amount of upward thrust. As in the initial stage, the rocket has to struggle against the gravity and the atmosphere. But the problem with solid fuel is that it can't be controlled. Other problem with solid fuel is that its specific impulse is low. Specific impulse is the impulse or change in momentum per unit mass of the fuel. Second stage is liquid fuel which imparts periodic kicks on the rocket at its perigee. Final stage is cryogenic stage. Here fuel is liquid hydrogen and liquid oxygen as the oxidant. Now the boiling point of oxygen is -183°C and that of hydrogen is −253°C respectively. These gases are stored in liquid state to make the matter compact and dense so that ample amount of propellant can be stored in the optimum allowed space.

Theme 1.0 Chandrayaan Utsav

1.1	F	हमारा चंद्रयान Our Chandrayaan
1.2	P	*Mera Pyara Chanda: Rani ki Khoj*
1.3	M	Chandrayaan Mission: Bharat’s Expedition to the Moon
1.4	S	Chandrayaan: Journey Towards the Moon
1.5	S	Exploring the Moon Mission of Bharat
1.6	S	Towards Moon and Beyond
1.7	S	Exploring Chandrayaan-3: Bharat’s Lunar Mission
1.8	HS	Bharat on the Moon
1.9	HS	Bharat Space Mission: The Chandrayaan Mission
1.10	HS	Physics of Chandrayaan-3

For participation in the activities related to Apna Chandrayaan:
Visit : www.bharatonthemoon.ncert.gov.in

For more information:
Email: dceta.ncert@nic.in
PMeVIDYA IVRS: 8800440559



An image of Rover *Pragyan* with Lander *Vikram*

विद्यया ऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING